

# How can one probe Podolsky Electrodynamics?\*

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## Abstract

We investigate the possibility of detecting the Podolsky generalized electrodynamics constant  $a$ . First we analyze an ion interferometry apparatus proposed by B. Neyenhuis, et al (Phys. Rev. Lett. 99, (2007) 200401) who looked for deviations from Coulomb's inverse-square law in the context of Proca model. Our results show that this experiment has not enough precision for measurements of  $a$ . In order to set up bounds for  $a$  we investigate the influence of Podolsky's electrostatic potential on the ground state of the Hydrogen atom. The value of the ground state energy of the Hydrogen atom requires Podolsky's constant to be smaller than 5.6 fm, or in energy scales larger than 35.51 MeV.

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## I. INTRODUCTION

The inference of the mass of the particles is a key problem in Physics. The Higgs mechanism is the most simple and popular way to generate massive particles from an originally gauge invariant massless theory. From the theoretical point of view the existence of a massive photon, usually considered in the context of Proca model, has many implications. One of the most important is the fact that interactions between particles are commonly described in terms of gauge theories and, as it is well known, the gauge field is supposed to be massless [1]. Since the electromagnetic interactions are described in terms of the  $U(1)$  symmetry group, all Quantum Electrodynamics, which is constructed on a gauge framework, should be reviewed if a mass for the photon was verified. The same occurs for instance in Atomic Physics, where the energy spectrum is supposed to be different if a non-Coulomb potential is considered.

Although it is widely accepted by physicists (especially by the theoreticians) that the photon is a massless particle, this is not an affirmation that can be easily done from the experimental point of view since all experiments are subject to uncertainties – the experimentalists basically establish upper limits for the photon mass.

Many experiments have been proposed to measure the mass of the photon [2] and among them, several try to accomplish this by using the fact that the electric field produced by a point charge is not the one predicted by Coulomb law if the photon is supposed to be massive. They try to verify the existence of a photon mass by looking for small deviations from the Coulomb law [3] – usually a potential  $1/r^{1+\delta}$  is tested, and  $\delta$  is evaluated. However, as mentioned in [5], the problem with this type of potential is that it does not come from any underlying theory and usually many assumptions regarding the measurement of  $\delta$  are done, so that its evaluation is strongly dependent on these hypothesis. In order to avoid these problems the authors of [5] proposed an experiment where an ion interferometry is used to measure the photon mass. The idea of the experiment consists, roughly speaking, in using interferometry of an ion beam that passes through a tube where different voltages are applied – if the mass of the photon is non-null then a difference in the interferometer phase is expected. According to the authors of [5], the experiment will be very accurate, predicting a sensitivity to the (Proca) mass of  $9 \times 10^{-50}$  g, “2 orders of magnitude smaller than the limit in [4]”. In their case the underlying theory is the Proca model.

However, if instead of using the Proca model, the Podolsky Generalized Electrodynamics [6] is taken into account, it is still possible to find a mass for the (massive mode of the) photon and preserve gauge symmetry. In a recent paper [7], a gauge theory for systems depending on the second

order derivative of the gauge field was developed and it was verified that the gauge Lagrangian should depend on the usual field strength,  $F_{\mu\nu}^a$ , and on its covariant derivative,  $G_{\rho\mu\nu}^a = D_{\rho b}^a F_{\mu\nu}^b$ . In particular, for the  $U(1)$  group it was verified that the Podolsky Lagrangian<sup>1</sup>,

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{a^2}{2}\partial_\rho F^{\rho\mu}\partial_\sigma F_\mu^\sigma, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

fulfills all the requirements of a second order gauge theory with an important feature: all Lagrangians of the type  $G^2$  for the  $U(1)$  group differs from Podolsky Lagrangian only by a total divergence. The (fourth-order) field equations obtained from this Lagrangian are

$$(1 + a^2\Box)\partial_\mu F^{\nu\mu} = 0,$$

and under a generalized Lorenz condition [8],  $(1 + a^2\Box)\partial_\mu A^\mu(x) = 0$ , massive and massless modes for  $A_\mu$  are identified:

$$p^2(1 - a^2p^2)A_\mu(p) = 0 \Rightarrow \begin{cases} p^2 = 0 \\ p^2 - \frac{1}{a^2} = 0 \end{cases}.$$

The massless mode should be understood as the usual photon, while the massive mode was tentatively interpreted by Podolsky as being a neutrino. This interpretation is of course outdated. Since its original formulation, several aspects of this theory have been analyzed, including its canonical structure [8, 9], quantization [10], and others [11]. Several problems of this theory have been pointed out, such as unitarity violation and the presence of ghost states with negative norm, typical of theories with higher derivatives [12], but on the other hand good properties were also obtained (see references in [10]), what motivates the study of systems of this kind nowadays, specially in the context of an effective field theory (EFT). It is as an EFT that Podolsky theory should be understood and in this sense the parameter  $a$  sets the length scale where the theory is valid. We also emphasize that only classical aspects of the Podolsky theory will be considered, so that some problems typical of the quantization procedure should not be a concern here.

Since Podolsky electrodynamics predicts the existence of a massive mode for the photon, if the experiment proposed in [5] finds a deviation in the interferometer phase, then this could be either an indicative of the existence of the photon mass in the context of the Proca model or of the existence of a non-null value for Podolsky constant, giving support to the Podolsky theory. One of the purposes of the present work is to analyze how the Podolsky constant can be determined

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<sup>1</sup> To preserve the correct units of the Lagrangian, the constant  $a$ , henceforth referred as the Podolsky constant, has unit  $\frac{1}{l}$ , where  $l$  stands for length; the metric signature  $(+ - - -)$  is considered.

or constrained by the ion interferometry experiment proposed in [5]. This is discussed in Section II, where the analytical solution for the problem will be analyzed and numerical estimations for Podolsky constant will be made.

On the other hand, if Podolsky theory is to be verified, then many implications in other known results are expected. As an example, the energy spectrum of the Hydrogen atom as described by Quantum Mechanics is to be altered, since the Coulomb potential should be substituted by the potential predicted by Podolsky Electrodynamics. This is the second point to be studied here. A perturbative solution for the Quantum Mechanics wave function of the electron will be obtained – see Section III – and another constraint on  $a$  will be made. Section IV presents our conclusions.

## II. ION INTERFEROMETRY EXPERIMENT

In the experiment proposed in [5] a time-varying voltage is applied to a conducting cylinder that is nested inside a grounded second cylinder. A beam of ions pass through the inner conductor through three gratings, forming a Mach-Zehnder interferometer – for more details see the original paper. If there is an electric field inside the cylinder, i.e. if the ions go through different potentials, then an interferometer phase shift is expected. Notice that this is not what is predicted by Maxwell equations for a conducting shell, according to which the potential inside the apparatus should be constant.

After passing through the first grating the ion beam is split in two arms: one travels horizontally (parallel to the cylinder axis), while the second goes diagonally. When the two arms reach the second grating, the one that was advancing horizontally begins to travel diagonally while the second starts to go horizontally, until they reach the third grating, where they become one single beam travelling horizontally. Since the distance between the gratings is the same, the diagonal segments of each arm travel through the same potentials and they induce the same phase shift. However, the segments of the arms that go horizontally pass through different potentials; if there is a phase shift in the interferometer it is caused by the difference of potentials between the horizontal segments (see Fig. 1). We consider that the distances of the horizontal segments from the center of the cylinder are  $r_0$  and  $r_0 + s$ . This way, what the interferometer actually does is to measure a phase shift induced by the potential difference between these horizontal segments of arms the split beam.

The first information required is the equation for the potential inside the cylinder as predicted by the theory. In [5] the authors considered the Proca model. Here we will analyze Podolsky

Electrodynamics [6], where the equation for the electrostatic potential is given by

$$(1 - a^2 \nabla^2) \nabla^2 \phi = 0.$$

To solve this equation, let us define

$$U \equiv \nabla^2 \phi.$$

First we solve the homogeneous equation for  $U$

$$(1 - a^2 \nabla^2) U = 0, \tag{1}$$

and then consider the non-homogeneous equation for  $\phi$ ,

$$\nabla^2 \phi = U_h, \tag{2}$$

where  $U_h$  is a solution of (1). In view of the symmetry of the problem, cylindrical coordinates are considered and no angular dependence is expected. Also, since the inner cylinder has an elongated geometry, the infinite tube approximation can be done and no longitudinal dependence exists. The solution for (1) is found under these assumptions, and Eq. (2) becomes

$$\frac{1}{r} \frac{d}{dr} \left( r \frac{d\phi}{dr} \right) = AI_0 \left( \frac{r}{a} \right) + BK_0 \left( \frac{r}{a} \right), \tag{3}$$

where  $I_0$  and  $K_0$  are the modified Bessel functions of the first and second kind.

The integration of Eq. (3) gives us

$$\phi \left( \frac{r}{a} \right) = a^2 AI_0 \left( \frac{r}{a} \right) + a^2 BK_0 \left( \frac{r}{a} \right) + D \ln \frac{r}{a} + C. \tag{4}$$

This solution carries a desirable feature: the homogeneous part is the usual Maxwell term and the particular solution is the Podolsky contribution. In fact, this split always occurs in the electrostatic case of Podolsky theory when vacuum is assumed.

Four integration constants appear in the solution (4), as expected from a fourth-order equation, and boundary conditions are used to fix them. First we consider that the potential in the limit  $r \rightarrow 0$  should be finite. Using the asymptotic form for  $I_0$  and  $K_0$  [13, 14], we conclude that  $D = a^2 B$ . Another boundary condition that is used is the value of the potential at  $r = R$ , where  $R$  is the radius of the inner tube. If  $V_0$  is the voltage applied to the inner tube relative to the outer tube, whose unknown (ground) potential is  $V_g$ , then

$$V_0 + V_g = a^2 A \left[ I_0 \left( \frac{R}{a} \right) + g(a) \left[ K_0 \left( \frac{R}{a} \right) + \ln \frac{R}{a} \right] + f(a) \right],$$

where  $B$  and  $C$  were redefined as  $B = g(a)A$  and  $C = f(a)a^2A$ , and  $A$  is supposed to be non-null. This expression is used to determine  $A$  in terms of the other constants. Yet another expected boundary condition is that the electric field  $\mathbf{E}$  at  $r = 0$  is null (otherwise it would be discontinuous without a physical reason). Actually with the redefinitions of  $B$  and  $C$  above, it is verified that this condition is already satisfied, so that no other constant is fixed with this condition. However, if we claim that the divergent of the electric field is finite at  $r = 0$ ,<sup>2</sup> then we must set  $g(a) = 0$ . At last, in order to fix  $f(a)$  we assume that the potential at  $r = 0$  can be measured – this is an additional step in the experimental procedure proposed in [5] where no measurement of the potential at  $r = 0$  is suggested; in our case this is essential for determining the last integration constant. We suppose that the measured  $\phi(0)$  is expressed as  $\phi(0) = (V_0 + V_g)\epsilon$ , with  $0 \leq \epsilon \leq 1$ . This fixes  $f(a)$  as

$$f(a) = \frac{\epsilon I_0\left(\frac{R}{a}\right)}{(1 - \epsilon)}.$$

Finally the potential is written as

$$\phi\left(\frac{r}{a}\right) = (V_0 + V_g) \left[ \frac{I_0\left(\frac{r}{a}\right)}{I_0\left(\frac{R}{a}\right)} (1 - \epsilon) + \epsilon \right].$$

Notice that if no Podolsky term is supposed to exist, then the potential inside the inner tube will be the same everywhere, i.e.  $V_0 + V_g$ , which means that  $\epsilon = 1$ .

Now the potential difference between the horizontal segments of the arms of the split beam can be evaluated as

$$\Delta\phi = \phi\left(\frac{r_0 + s}{a}\right) - \phi\left(\frac{r_0}{a}\right) = (V_0 + V_g) \left[ \frac{I_0\left(\frac{r_0 + s}{a}\right) - I_0\left(\frac{r_0}{a}\right)}{I_0\left(\frac{R}{a}\right)} \right] (1 - \epsilon).$$

The interferometer phase is given by

$$\Phi = \frac{e\tau}{\hbar} \Delta\phi + \Phi_0 = \frac{e\tau}{\hbar} (V_0 + V_g) \left[ \frac{I_0\left(\frac{r_0 + s}{a}\right) - I_0\left(\frac{r_0}{a}\right)}{I_0\left(\frac{R}{a}\right)} \right] (1 - \epsilon) + \Phi_0,$$

where  $e$  is the charge of the ion (in the present case  $e$  is the electron charge),  $\tau$  is the time that the ion takes to travel lengths of the horizontal segments and  $\Phi_0$  is the phase indicated by the interferometer when  $V_0 + V_g = 0$ . In order to eliminate the two unknown constants  $\Phi_0$  and  $V_g$ , two potential differences  $V_0$  and  $V_0 + \Delta V$  can be applied to the inner tube. The difference in the phases due to this change will be

$$\Delta\Phi = \frac{e\tau}{\hbar} \Delta V \left[ \frac{I_0\left(\frac{r_0 + s}{a}\right) - I_0\left(\frac{r_0}{a}\right)}{I_0\left(\frac{R}{a}\right)} \right] (1 - \epsilon).$$

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<sup>2</sup> What makes the electric field flux finite at the origin.

This expression is inverted in order to obtain the Podolsky constant  $a$  as a function of the experimental parameters. This will be done under some assumptions. First we expect that the value of Podolsky constant is small, so that only small differences from Maxwell equations can be detected. If this is the case, then the asymptotic limit for  $I_0$  can be used [13, 14],  $I_0(x) \sim \frac{1}{\sqrt{2\pi x}} e^x$ . This allows us to estimate the Podolsky constant as

$$a = \frac{R - (r_0 + s)}{\ln(1 - \epsilon) - \ln\left(\frac{\hbar}{e\tau} \frac{\Delta\Phi}{\Delta V} \sqrt{\frac{r_0 + s}{R}}\right)}. \quad (5)$$

Notice that  $\lim_{\epsilon \rightarrow 1} a = 0$ , which means that Electrodynamics reduces to the Maxwell one.

We shall obtain numerical estimations for  $a$  considering ion beams composed by  $^1H^+$  and  $^{133}Cs^+$ . According to [5], these ions can travel at a speed  $v$  of  $311 \text{ m/s}$  and  $27 \text{ m/s}$  respectively; the length of the horizontal segments are fixed in  $1 \text{ m}$  so that  $\tau = L/v$  is determined for both cases. The potential difference  $\Delta V$  can be fixed as  $400 \text{ kV}$  and the values of  $R$ ,  $r_0$  and  $s$  are set to  $R = 27 \text{ cm}$ ,  $r_0(^1H^+) = 24.4 \text{ cm}$ ,  $r_0(^{133}Cs^+) = 24.9 \text{ cm}$ , and  $s(^1H^+) = 6.4 \text{ mm}$ ,  $s(^{133}Cs^+) = 0.56 \text{ mm}$ . Fig. 2 shows the numerical estimations for  $a$  for different values of  $\epsilon$  and  $\Delta\Phi$  for  $^1H^+$ . The range of values for  $\epsilon$  was established considering the fact that a precision of  $10^{-8}$  could be achieved with the available commercial multimeters (in the best case). Concerning  $\Delta\Phi$ , it was considered that phase shifts as small as  $10^{-4} \text{ rad}$  can be detected [5].

According to these numerical evaluations, the experiment would be able to detect values of the Podolsky constant  $a_{Cs^+} \geq 0.033 \text{ cm}$  in the case of  $^{133}Cs^+$  ion beam and  $a_{H^+} \geq 0.069 \text{ cm}$  if the  $^1H^+$  ion beam is used. These values seem consistent with the asymptotic limit taken for  $I_0$ ; indeed, they are small when compared to the values of  $R$  and  $r_0$  and therefore the ratios that appear in  $I_0$  – namely,  $\frac{R}{a_{H^+}} = 391.3$ ,  $\frac{R}{a_{Cs^+}} = 810.81$ ,  $\frac{r_0}{a_{H^+}} = 353.62$ ,  $\frac{r_0}{a_{Cs^+}} = 732.73$  – are all of order of  $10^2 - 10^3$ .

The mass of the photon is evaluated using these values for  $a$  and the expression

$$m_\gamma = \frac{\hbar}{ac}. \quad (6)$$

As the mass scales with the inverse of the Podolsky constant, the smallest value of  $a$  that can be measured will give the greatest measurable value for the photon mass. Each ion beam will predict a different upper limit:  $m_\gamma^{^{133}Cs^+} = 1.06 \times 10^{-39} \text{ kg} = 5.98 \times 10^{-8} \text{ eV}$  and  $m_\gamma^{^1H^+} = 5.10 \times 10^{-40} \text{ kg} = 2.85 \times 10^{-8} \text{ eV}$ .

Although Proca and Podolsky approaches predict a massive mode for the photon, there is some important difference between them. First, Podolsky Electrodynamics is a gauge theory, while Proca model explicitly break such symmetry, what could have implications for the charge conservation.

Second, in the Proca context it is expected that the photon mass, if it exist, should be very small. Conversely, the Podolsky's massive model would be very large once it is the inverse of the scale of length where the generalized theory is effective, cf. Eq. (6). That is, Proca (Podolsky) model predicts deviations from Maxwell electrodynamics in very low (high) energy regimes.

It is important to emphasize that the photon mass is independent of the nature of the ion composing the beam in the experiment. The different values for  $m_\gamma$  for  $^{133}\text{Cs}^+$  and  $^1\text{H}^+$  express only the different values of  $a$  accessed by the experiment.

One could argue that the values of  $a$  that can be measured by the ion interferometer are very high in absolute terms. In fact, one would say that if  $a$  were of order of  $10^{-2}$  as indicated here, the deviations from the Maxwellian electromagnetism would have been detected long ago. In face of this, the conclusion would be that the experiment proposed in [5] is not appropriate for measuring the Podolsky constant and therefore the photon mass in this theory. This is indeed a strong argument, but we would like to give a quantitative justificative for ruling out the ion beam apparatus as an appropriate set to find the Podolsky mass.

In the next section we will make the hypothesis that Podolsky electrodynamics hold at the atomic scale<sup>3</sup> and see the implications for the elementary physics of the Hydrogen atom; in particular, we will analyze the energy of the fundamental state.

### III. HYDROGEN ATOM

Now we turn to the problem of considering the Hydrogen atom, as treated by Quantum Mechanics, where the electromagnetic potential is the one described by Podolsky Electrodynamics. The goal of this section is to analyze the effects of a non-null Podolsky constant and verify what are the implications of the values found for  $a$ . We consider  $\hbar = 1$  to simplify the notation, but the units are restored when numerical evaluations are done.

The electrostatic potential is given by [6, 7]

$$\phi(r) = -\frac{e}{r} \left(1 - e^{-\frac{r}{a}}\right),$$

and the Hamiltonian operator reads  $\hat{H} = \frac{\hat{p}^2}{2m} + e\phi(r)$ . The variational method will be employed so that a perturbative solution for the wave function of the ground state,  $\psi(r)$  may be found. The tentative solution is

$$\psi(r) = Ne^{-\gamma r},$$

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<sup>3</sup> This is not mandatory once Podolsky's theory for the electromagnetism is an effective theory.



where  $N$  is a normalization constant set as  $N^2 = \frac{\gamma^3}{\pi}$ ;  $\gamma$  is a parameter that will be determined by the variational method, according to which the energy, given by

$$E = \int dV \psi^*(r) \hat{H} \psi(r) = \frac{\gamma^2}{2m} - e^2 \gamma + e^2 \frac{4\gamma^3}{(2\gamma + \frac{1}{a})^2},$$

should be minimized:

$$\frac{\partial E}{\partial \gamma} = \frac{8a^3}{m} \gamma^4 + \frac{12a^2}{m} \gamma^3 + \frac{6a}{m} \gamma^2 - 6ae^2 \gamma + \frac{\gamma}{m} - e^2 = 0. \quad (7)$$

Now suppose that the value of the Podolsky constant is actually small, then Eq. (7) can be solved considering only terms up to first order in  $a$ . The solution found for  $\gamma$  is  $\gamma_+ = me^2$  and  $\gamma_- = -\frac{1}{6a}$ . The energies evaluated with these solutions are

$$E(\gamma_+) = -\frac{me^2}{2} e^2 \left(1 - 2(2mae^2)^2\right) + O(a^3), \quad E(\gamma_-) = \frac{9ame^2 + 1}{72a^2m}.$$

The value of  $E(\gamma_-)$  gives a positive energy and for small values of  $a$  it becomes too high, therefore this result should be excluded.  $E(\gamma_+)$  can only be calculated with a given value of  $a$ , but for small  $a$  it is only a perturbation on the known result given by Quantum Mechanics,  $E = -\frac{me^2}{2} e^2$ . If we want to find a value for  $a$  that is compatible with the known results given in the literature we should expect the perturbation  $2(2mae^2)^2$  to be smaller than the relative experimental uncertainty of the energy of the ground state. Proceeding this way it follows

$$a \leq \frac{r_B}{2} \sqrt{\frac{\sigma_{E_0}}{2|E_0|}},$$

where  $r_B = \frac{1}{me^2}$  is the Bohr radius. Restoring the units and using values given in the literature [15] we should expect

$$a \leq 5.56 \text{ fm} \quad \text{or} \quad m_\gamma \geq 35.51 \text{ MeV} \quad (8)$$

Clearly these values for  $a$  and  $m_\gamma$  are not compatible with the possible values that can be found in the interferometry experiment.

#### IV. CONCLUSIONS

We have discussed how the ion interferometry experiment proposed in Ref. [5] could be used to measure the value of Podolsky constant  $a$  and the massive mode of the photon in the context of Podolsky Electrodynamics. The minimum value of  $a$  that could be detected –  $a = 0.033 \text{ cm}$  with the  $^{133}\text{Cs}^+$  ion beam – is too large as an admissible effective scale, and would lead to a mass

$m_\gamma \leq 1.06 \times 10^{-39} \text{ kg} = 5.98 \times 10^{-8} \text{ eV}$  for the photon which is excluded by current experimental data [15].

We might think of improving the accuracy of the measurements of the phase shift and/or of the potential at  $r = 0$  (for instance, using some better technology in the apparatus). But the logarithmic behavior of (5) in terms of these quantities makes this possibility unlikely: great improvements in the detection of  $\Delta\Phi$  and  $\Delta V$  would lead to small changes in  $a$  [see Eq. (5)]. Therefore, this rules out the interferometric ion beam experiment as a suitable one for testing Podolsky Electrodynamics.

Besides gauge invariance, Podolsky Electrodynamics has another peculiar feature that distinguishes it from the Proca field: the smaller the characteristic constant  $a$  the greater the mass associated to the photon. Hence we are strongly constrained: the Maxwellian electromagnetism must hold until small scales of length, and therefore  $a$  has to be small, otherwise the additional Podolsky term in the Lagrangian for the electromagnetic field would be significant and the resulting modifications in the ordinary theory would be easily detected. These scales of length are set, for instance, by the spectroscopy of Hydrogen atom. So, we tested Podolsky's theory calculating the value of  $a$  that would be consistent with the experimental error in the energy of the fundamental level of the Hydrogen. The result,  $a \leq 5.56 \text{ fm}$ , clearly shows that the ion interferometer experiment does not have enough precision to measure a Podolsky constant that is this small.

The constraint  $a \leq 5.56 \text{ fm}$  coming from Quantum Mechanics considerations set a high energy scale for the photon mass:  $m_\gamma > 35.51 \text{ MeV}$ . This way, if Podolsky model is correct, it is expected to engender deviations from Maxwell Electrodynamics only in high energy scales, which are accessible by particle accelerators. Therefore, the next necessary step is to investigate in more detail which kind of effects appear in QED<sub>4</sub> due to the Podolsky term.

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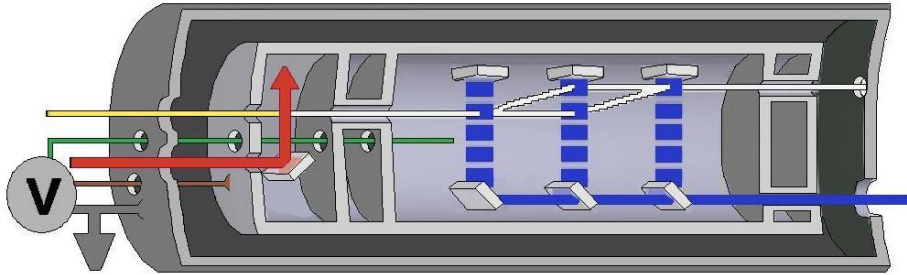


FIG. 1: Sketch of the ion interferometry experiment. A cutaway of the cylinders is shown.

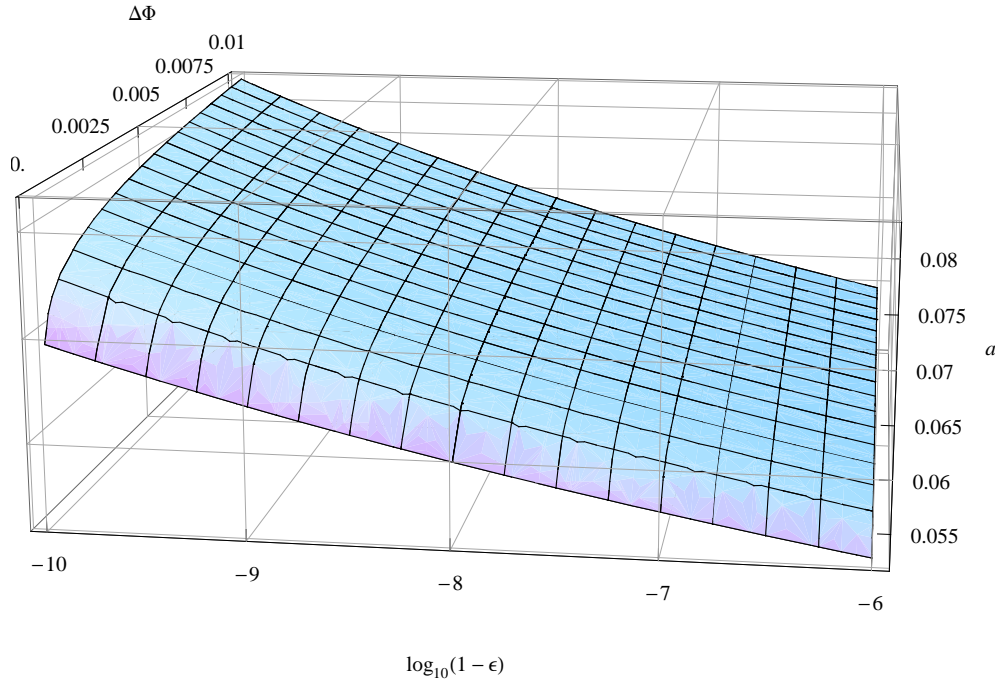


FIG. 2: Values of  $a$  (cm) for different values of  $\epsilon$  (from 0.001 to 0.999) and  $\Delta\Phi$  (rad) (from  $10^{-4}$  to  $10^{-2}$ ) using  $^1H^+$  ion beam. The graph for the  $^{133}Cs^+$  ion beam is very similar.